

Index of notation

Entries are listed in order of appearance.

∞ : arithmetic with infinity, xiv	$\mathbb{P}[\mathcal{A} \mid \mathcal{B}]$: conditional probability of \mathcal{A} given \mathcal{B} , 100
\log : natural logarithm, xiv	$\mathbb{E}[X]$: expected value of X , 111
\exp : exponential function, xiv	$\text{Var}[X]$: variance of X , 113
\emptyset : the empty set, xiv	$\mathbb{E}[X \mid \mathcal{B}]$: conditional expectation of X given \mathcal{B} , 114
$A \cup B$: union of two sets, xiv	$\Delta[X; Y]$: statistical distance, 131
$A \cap B$: intersection of two sets, xiv	mG : $\{ma : a \in G\}$, 185
$A \setminus B$: difference of two sets, xiv	$G\{m\}$: $\{a \in G : ma = 0_G\}$, 186
$S_1 \times \dots \times S_n$: Cartesian product, xv	G^m : $\{a^m : a \in G\}$, 186
$S^{\times n}$: n -wise Cartesian product, xv	$H_1 + H_2$: $\{h_1 + h_2 : h_1 \in H_1, h_2 \in H_2\}$, 189
$f(S)$: image of a set, xv	$H_1 \cdot H_2$: $\{h_1 h_2 : h_1 \in H_1, h_2 \in H_2\}$, 189
$f^{-1}(S)$: pre-image of a set, xv	$a \equiv b \pmod{H}$: $a - b \in H$, 190
$f \circ g$: function composition, xvi	$a + H$: coset of H containing a , 190
\mathbb{Z} : the integers, 1	aH : coset of H containing a (multiplicative notation), 190
$b \mid a$: b divides a , 1	G/H : quotient group, 191
$\lfloor x \rfloor$: floor of x , 3	$[G : H]$: index, 191
$a \bmod b$: integer remainder, 3	$\ker(\rho)$: kernel, 194
$\lceil x \rceil$: ceiling of x , 3	$\text{img}(\rho)$: image, 194
$a\mathbb{Z}$: ideal generated by a , 4	$G \cong G'$: isomorphic groups, 197
$a_1\mathbb{Z} + \dots + a_k\mathbb{Z}$: ideal generated by a_1, \dots, a_k , 5	$\langle a \rangle$: subgroup generated by a , 202
\gcd : greatest common divisor, 6	$\langle a_1, \dots, a_k \rangle$: subgroup generated by a_1, \dots, a_k , 202
$\nu_p(n)$: largest power to which p divides n , 8	\mathbb{R} : real numbers, 212
lcm : least common multiple, 9	\mathbb{C} : complex numbers, 212
\mathbb{Q} : the rational numbers, 9	$\bar{\alpha}$: complex conjugate of α , 212
$a \equiv b \pmod{n}$: a congruent to b modulo n , 13	$N(\alpha)$: norm of $\alpha \in \mathbb{C}$, 213
$b/a \bmod n$: integer remainder, 17	$b \mid a$: b divides a , 214
$a^{-1} \bmod n$: integer modular inverse, 17	R^* : multiplicative group of units of R , 214
\mathbb{Z}_n : residue classes modulo n , 21	$\mathbb{Z}[i]$: Gaussian integers, 219
ϕ : Euler's phi function, 24	$\mathbb{Q}^{(m)}$: $\{a/b : \gcd(b, m) = 1\}$, 219
μ : Möbius function, 29	$R[X]$: ring of polynomials, 220
$O, \Omega, \Theta, o, \sim$: asymptotic notation, 33	$\deg(a)$: degree of a polynomial, 223
len : length (in bits) of an integer, 46	$\text{lc}(a)$: leading coefficient of a polynomial, 223
$\text{rep}(\alpha)$: canonical representative of $\alpha \in \mathbb{Z}_n$, 48	$a \bmod b$: polynomial remainder, 224
$\pi(x)$: number of primes up to x , 74	$\mathbf{D}(a)$: formal derivative of a , 227
ϑ : Chebyshev's theta function, 76	
li : logarithmic integral, 87	
ζ : Riemann's zeta function, 88	
P : probability function, 96	

- $a_1 R + \cdots + a_k R$: ideal generated by
 a_1, \dots, a_k , 231
- (a_1, \dots, a_k) : ideal generated by a_1, \dots, a_k , 231
- R/I : quotient ring, 232
- $[a]_I$: the coset $a + I$, 232
- $[a]_d$: the coset $a + dR$, 232
- $R \cong R'$: isomorphic rings, 237
- $\log_\gamma \alpha$: discrete logarithm, 268
- $(a | p)$: Legendre symbol, 285
- $(a | n)$: Jacobi symbol, 287
- J_n : Jacobi map, 289
- aM : $\{a\alpha : \alpha \in M\}$, 301
- $M\{a\}$: $\{\alpha \in M : a\alpha = 0_M\}$, 301
- $\langle \alpha_1, \dots, \alpha_n \rangle_R$: submodule spanned by
 $\alpha_1, \dots, \alpha_n$, 302
- $R[\mathbf{x}]_{<\ell}$: polynomials of degree less than ℓ , 302
- M/N : quotient module, 303
- $M \cong M'$: isomorphic modules, 304
- $\dim_F(V)$: dimension, 311
- $A(i, j)$: (i, j) entry of A , 317
- $A(i)$: i th row of A , 317
- $A(\cdot, j)$: j th column of A , 317
- $R^{m \times n}$: $m \times n$ matrices over R , 317
- A^\top : transpose of A , 319
- $\Psi(y, x)$: number of y -smooth integers up to x ,
336
- gcd: greatest common divisor (polynomial),
368
- lcm: least common multiple (polynomial), 370
- $b/a \bmod n$: polynomial remainder, 371
- $a^{-1} \bmod n$: polynomial modular inverse, 371
- $(E : F)$: degree of an extension, 377
- $R[[\mathbf{x}]]$: formal power series, 379
- $R((\mathbf{x}))$: formal Laurent series, 380
- $R((\mathbf{x}^{-1}))$: reversed formal Laurent series, 381
- $\deg(a)$: degree of $a \in R((\mathbf{x}^{-1}))$, 381
- $\text{lc}(a)$: leading coefficient of $a \in R((\mathbf{x}^{-1}))$, 381
- $\lfloor a \rfloor$: floor of $a \in R((\mathbf{x}^{-1}))$, 382
- len: length of a polynomial, 399
- $\text{rep}(\alpha)$: canonical representative of
 $\alpha \in R[\mathbf{x}]/(n)$, 400
- $\mathcal{D}_F(V)$: dual space, 429
- $\mathcal{L}_F(V)$: space of linear transformations, 440
- $\mathbf{N}_{E/F}(\alpha)$: norm, 458
- $\mathbf{Tr}_{E/F}(\alpha)$: trace, 458

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